Geometric Method to obtain Harmonic Mean of Two numbers

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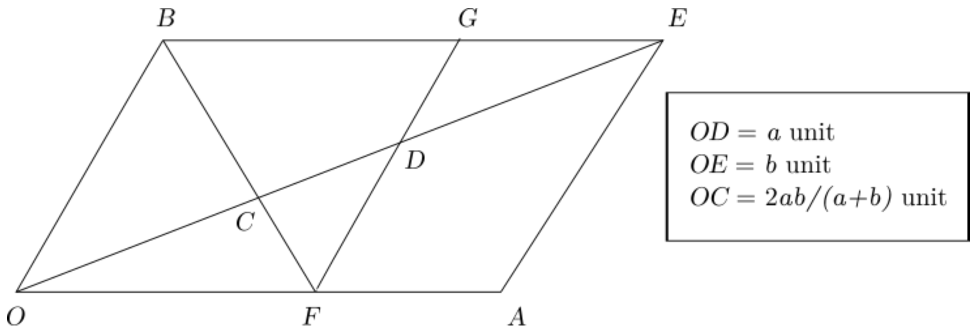

Let the two numbers whose harmonic mean has to be determined be a and b ($b \geq a$).
Any parallelogram $OAEB$ is drawn, whose diagonal $OE = b$ unit.
D is a point on OE such that $OD = a$ unit.
$FG \parallel AE$ is drawn through D.
BF is drawn, which intersects OE at C.
Then, the magnitude of twice the length of OC is the harmonic mean of a and b.

Proof:

$\frac{OC}{CD} = \frac{OB}{FD}$ ($\because \triangle OBC$ and $\triangle CFD$ are similar)

$\therefore \frac{OC}{CD} = \frac{AE}{FD}$($\because OE = FD$) ————————————————(1)

Also, $\frac{OC}{CD} = \frac{FD}{AE}$ ($\because \triangle OFD$ and $\triangle OAE$ are similar) ————(2)

From (1) and (2), $\frac{OC}{CD} = \frac{OE}{OD}$

Let $OC = x$ unit. $\therefore CD = (a - x)$ unit.

Thus, $\frac{x}{a-x} = \frac{b}{a}$

$\implies x = \frac{ab}{a+b}$

Therefore, twice the magnitude of OC is the harmonic mean of a and b.

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